Math 410/610 Homework 10

Due on Nov 28

1. Show that
\[ \int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2 \theta} = \sqrt{2\pi}. \]

2. Show that
\[ \int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx = \pi. \]

(Hint: consider integrating \( \frac{e^{\pm 2iz} - 1}{z^2} \) over appropriate indented contour)

3. Show that
\[ \int_{0}^{\infty} \frac{\ln x}{x^3 - 1} dx = \frac{4\pi^2}{27}. \]

(Hint: Integrate around the boundary of an indented sector of aperture \( \frac{2\pi}{3} \))