Math 410/610 Homework 8

Due on Nov 6

1. Find the isolated singularities on $\mathbb{C}^*$ of the following functions, and determine whether they are removable, poles or essential. Determine the order of any pole, and find the principal part at each pole.
   (a) $\frac{z+5}{(z^2-1)^2}$
   (b) $z^3 \sin(1/z)$

2. A meromorphic function $f$ at $z_0$ is said to have order $N \in \mathbb{Z}$ at $z_0$ if $f(z) = (z - z_0)^N g(z)$ for some function $g$ analytic at $z_0$ such that $g(z_0) \neq 0$. The order of the function 0 is defined to be $+\infty$. Show that
   (a) $\text{order}(fg, z_0) = \text{order}(f, z_0) + \text{order}(g, z_0)$
   (b) $\text{order}(1/f, z_0) = -\text{order}(f, z_0)$

3. Show that if $z_0$ is an isolated singularity of $f(z)$ that is not removable, then $z_0$ is an essential singularity for $e^{f(z)}$. 