1 Suppose that \( f(z) \) is an entire function and \( |f''(z)| \leq 5 \) for all \( z \in \mathbb{C} \). Show that \( f \) must be of the form \( az^2 + bz + c \) for some constants \( a, b, c \in \mathbb{C} \).

2 The entire function \( \sin(z) \) is equal to 0 at the infinite sequence of points \( 0, \pi, 2\pi, 3\pi, \cdots \). Can you use the Uniqueness Principle of analytic functions to conclude that \( \sin(z) \equiv 0 \)? Explain why.

3 Let \( f(z) = \frac{1}{3 - z + z^2} \).
   (a) Find the power series expansion of \( f \) around \( z = 0 \).

   (b) Find the Laurent series expansion of \( f \) around \( z = 1 \).
4 Determine the domain of convergence for the Laurent series \[ \sum_{n=\infty}^{-1} \frac{z^n}{n^2} + \sum_{n=0}^{\infty} (z/3)^n. \]

5 Suppose that \( f \) has an isolated singularity at 0 and satisfies \( |f(z)| \leq \frac{1}{|z|} \) in some deleted neighborhood of 0. Show that \( f \) has a removable singularity or a simple pole at 0.

6 Show that if \( z_0 \) is an isolated singularity of \( f(z) \), and if \( (z - z_0)^N f(z) \) is bounded in some punctured neighborhood of \( z_0 \) for some positive integer \( N \), then \( z_0 \) is either removable or a pole of order at most \( N \).

7 Show that if \( z_0 \) is an isolated singularity of \( f(z) \), and if \( \lim_{z \to z_0} (z - z_0)f(z) = 0 \), then \( z_0 \) is removable.
8 Find all the singularities of the function \( \frac{(2z-4)e^{(z-1)}}{z^3-4z} \) and identify their types.

9 Use the Residue Theorem to evaluate
\[
\int_{|z|=2} \frac{dz}{(5-z^2)(z^4+1)}.
\]

10 Evaluate the improper integral
\[
\int_{-\infty}^{\infty} \frac{dx}{1+x^2+x^4}.
\]