STATS 758, Spring 2012

Midterm 2 (April 16, 2012)

Name: ________________________________

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- Read assignments carefully
- If you do not see a solution — go to the next problem, good ideas may come later
- Write answers in the space provided
- Show work wherever it is possible

- **Good Luck!**
Problem 1 [15 points] Give an example of SARIMA process of order

\[(p, d, q) \times (P, D, Q)_s = (0, 1, 2) \times (1, 1, 0)_{12}.\]
Problem 2 [25 points] For the model $X_t = 0.2 X_{t-1} + Z_t$ with $Z_t \sim WN(0, 0.96)$

a) [5 points] Classify the model as ARIMA($p, d, q$) (that is, find $p$, $d$, and $q$).

b) [5 points] Check stationarity, causality, invertibility.

c) [5 points] Find the first three $\pi$-weights in AR($\infty$) representation.

d) [5 points] Find the first three $\psi$-weights in MA($\infty$) representation.

e) [5 points] Find the variance of $X_t$. 
Problem 3 [20 points] Consider process $X_t = 0.7 X_{t-1} + 0.1 X_{t-2} + Z_t$, where $Z_t \sim \text{WN}(0, \sigma^2)$.

a) [10 points] Construct a 1-step forecast for $X_t$.

b) [10 points] Find the variance of the forecast error.
Problem 4 [20 points] Find the range of $\alpha$ for which the following process is stationary, causal, and invertible:

$$X_t = 3\alpha Z_t - (3 + \alpha)Z_{t-1} + Z_{t-2}.$$
Problem 5 [20 points] The file Midterm2.txt from the course web site contains a time series from an ARIMA model.

   a) [15 points] Identify the model’s order \((p, d, q)\).

   b) [5 points] Estimate model’s coefficients.